Kocherlakota (RES 1996)
Implications of Efficient Risk Sharing without Commitment

- Consumption data indicates that risk sharing is imperfect: conditional con per capita consumption, individual consumption depends on current individual income and lagged incomes as well. In particular, they covariate in a positive way.
- This is not what would happen in a frictionless economy.
- Several papers give a rationale for this. It is usually assumed that the friction is assymetric information (and costly monitoring).
- Kocherlakota argues that this argument is not quite appealing: within a risk-sharing pool, informational assymetries are small.
- Kocherlakota abstracts from assymetric information and explores whether this can be explained by lack of commitment.
- He models lack of commitment by using repeated games techniques: lack of commitment means that I cannot ex-ante commit to a given strategy, but rather, at any point in time, given the history up to that moment and the strategies of the other playes, I will behave in a rational (expected utility maximizing) way.
- **Environment**
  - Time is discrete.
  - The economy is populated by two infinitely-lived agents.
  - At each period t, the state of the world is determined by the realization of a discrete iid r.v. that determines each individual's income.
  - There is a single perishable good.
  - The joint distribution of individual incomes (y_1, y_2) is symmetric.
  - Individual's preferences are identical: expected discounted utility, with common discount factor $\beta \in (0,1)$: ¨at period t they are described by

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} u\left(c_{t+\tau}\right).$$

  - u: increasing, strictly concave, continuously differentiable and $\lim_{c\to 0} u'(c) = \infty$ to rule out corner solutions.
- A consumption allocation $\left(\left(c_t^j\right)_{t=1}^{\infty}\right)_{j=1}^{2}$ is **feasible** if it is non-negative and $c_t^1 + c_t^2 \leq Y_t \ \forall t$.
- An allocation is **first-best** if $c_t^1 + c_t^2 = Y_t$ for all dates and states and $\frac{u'\left(c_t^1\right)}{u'\left(c_t^2\right)}$ is constant for all dates and states.

- So, conditional on aggregate output Y_t, c_t^j has to be constant over dates and states. In particular, it does not covary with individual income (current or lagged) $\rightarrow$ this is not what we see in the data.

- **The Dynamic Game**
  - Previous environment plus...
  - At each period t, after individuals observe both incomes, they simultaneously decide a non-negative transfer to make to the other agent.
  - A period$-t$ history is $(\theta_1, TR_1, \ldots, \theta_{t-1}, TR_{t-1}, \theta_t)$.
  - A period$-t$ strategy for each player is a function that maps from the set of possible period$-t$ histories to transfer amounts.
  - An SPE is a stratefy profile such that, after any possible history and given the agent 2 strategy, agent 1's action is optimal.

- Two results:
  - Autarky is the SPE that provides less utility to both agents.
  - A feasible allocation is SPE if and only if

$$u\left(c_t^j\right) + E_t\left[\sum_{\tau=0}^{\infty} \beta^\tau u\left(c_{t+\tau}^j\right)\right] \geq u\left(y_t^j\right) + \beta V_{aut}, \quad j = 1, 2$$

    for all dates and states.
  - (It is assumed that there exists a non-autarkic subgame-perfect allocation: a sufficient condition is $\exists s : \pi_s \leq 0.5$ and $[(1-\beta) + \beta\pi_s] u'\left(y_s^1\right) - \beta\pi_s u'\left(y_s^2\right) < 0$.)

- **The Pareto frontier of the set of SPE values**
  - Let $V_{\max}$ be the maximial level of utility to a given agent from an SPE allocation, and define the function $V : [V_{aut}, V_{\max}] \rightarrow [V_{aut}, V_{\max}]$ by

$$\begin{aligned} V(u_0) &= \max_{\{c_s, u_s\}_{s=1}^S} \sum_{s=1}^S \pi_s \left[u(Y_s - c_s) + \beta V(u_s)\right] \\ s.t. &: \left\{ \begin{array}{c} \sum_{s=1}^S \pi_s \left[u(c_s) + \beta u_s\right] = u_0 \\ u(c_s) + \beta u_s \geq u\left(y_s^1\right) + \beta V_{aut}, \forall s \\ u(Y_s - c_s) + \beta V(u_s) \geq u\left(Y_s - y_s^1\right) + \beta V_{aut}, \forall s \\ u_s \in [V_{aut}, V_{\max}] \end{array} \right. . \end{aligned} \quad (1)$$

  - V takes an element in $[V_{aut}, V_{\max}]$, which is the expected utility promised to agent 1 yesterday, and gives the highests expected utility that agent 2 can obtain if this promise is kept and the implicit allocations form an SPE.

- If an agent's sustainability constraint is binding, it is efficient to provide him with more utility in the future.

- If in some state s none is binding, then $u_s = u_0$ (the "same" situation).
- u_s is a non-decreasing function of c_s : agents tend to face binding sustainability constraints when their income shocks are high.
- There is a contemporaneus correlation between individual income and individual consumption.
- There is some persistence of high income shocks even if individual incomes are iid. $\rightarrow$ they can't smooth consumption over states because of lack of commitment, so they do it over time.
- **Result 1:** In an efficient allocation $Cov\left(c_t^1, y_{t-k}|Y_{t-k}\right) \geq 0$ for $k \geq 0$. For $k = 0$, if this conditional covariance is 0 for all realizations of Y_t, then the efficient allocation is first best in all ensuing dates and states.
- **Result 2:** When SPE first-best allocations exist, then if $u_0 > u^{FB}$, then with probability 1, u_t converges monotonically to u^{FB}. If $u_0 < u_{FB}$, then with probability one, u_t converges monotonically to u_{FB}.
  - When they are sufficiently patient, lack of commitment cannot justify the observed lack of diversification in individual consumption as being efficient.
- **Result 3:** When SPE first-best allocations do not exist, then $Cov\left(c_t^1, y_t|Y_t\right) > 0$. Also, not only the sign but the magnitude of the correlation is the ame in all efficient allocations: as t goes to infinity, $\Pr\left(u_t|u_0\right)$ converges weakly to the same non-degenerate limiting distribution for all $u_0 \in \left[V_{aut}, V_{\max}\right]$.
- How to test this theory against one with an assymetric-information/commitment environment?
  - The past is fully summarized by u_t, and in a very special way, since every efficient SPE satisfies (if $u_t \in (V_{aut}, V_{\max})$):

$$\gamma_t := \frac{u'\left(c_t^2\right)}{u'\left(c_t^1\right)} = -V'\left(u_t\right). \tag{2}$$

  - So: evolution of consumption depends only on the ratio of current marginal utilities.
  - In the other environment, there is generally no such a relation: consumption in other states shows up in IC constraints because of private information, so there is no direct linkage between γ_t and u_t. So, γ_t will not be a sufficient statistic.

$$\Pr\left(\left(c_{t+1}^n\right)_{n=1}^N |\gamma_t, Y_t, \left(c_\tau\right)_{\tau=0}^{t-1}\right) = \Pr\left(\left(c_{t+1}^n\right)_{n=1}^N |\gamma_t\right)$$