Consider the following dynamic programming problem:

$$\begin{aligned} v(a,y) &= \max_{(c,a')\in\Gamma(a,y)} u(c) + \beta E\left[v(a',y')\,|y\right] \\ \Gamma(a,y) &: = \left\{(c,a') \in \mathbb{R}_+ \times \mathbb{R} : c + a' \leq (1+r)\,a + y\right\}. \end{aligned} \tag{1}$$

Proposition 1 *If preferences exhibit prudence then savings increase after a mean preserving spread.*

Proof
Euler Equation is

$$u'\left((1+r)\,a + y - a'\right) = \beta(1+r)\,E\left[u'\left(y' + (1+r)\,a' - a''\right)|y\right]. \tag{2}$$

Now consider $\widetilde{y} = y + \varepsilon$ where $E[\varepsilon] = 0$ and $V(\varepsilon) > 0$. We want to show that

$$E\left[u'\left(y' + (1+r)\,a' - a''\right)|y\right] < E\left[u'\left(\widetilde{y}' + (1+r)\,a' - a''\right)|y\right]. \tag{3}$$

Let $f : \mathbb{R} \to \mathbb{R}$ be a strictly convex twice differentiable function and let's consider a $2nd$ order Taylor approximation of f around $E[x]$, where x is a random variable. We have that

$$f(x) \approx f(E[x]) + f'(E[x])\,(x - E[x]) + \frac{f''(E[x])}{2}\,(x - E[x])^2. \tag{4}$$

Now take expectations to obtain

$$E[f(x)] \approx f(E[x]) + \frac{f''(E[x])}{2} Var(x). \tag{5}$$

Since $f'' > 0$ by hypothesis, we have that $E[f(x)]$ increases with $Var(x)$ provided that $E[x]$ remains the same.

So, after a mean preserving spread we have

$$\begin{aligned} u'\left((1+r)\,a + y - \widetilde{a}'\right) &= E\left[u'\left(\widetilde{y}' + (1+r)\,a' - a''\right)|y\right] \\ &> E\left[u'\left(y' + (1+r)\,a' - a''\right)|y\right] \\ &= u'\left((1+r)\,a + y - a'\right), \end{aligned} \tag{6}$$

so

$$\widetilde{a}' > a'. \tag{7}$$